Singular Semi-Flat Calabi-Yau Metrics on S^2

John Loftin
Department of Mathematics
Columbia University
New York, NY 10027
USA

Abstract

If u is a convex function on a domain $\Omega \subset \mathbb{R}^n$ satisfying the Monge-Ampère equation $\det(u_{ij}) = 1$, then there is a natural Calabi-Yau metric on the tube domain $\Omega + i\mathbb{R}^n$: Extend u to be constant on the imaginary fibers and the Kähler metric $u_{i\bar{j}} dz^i \overline{dz^j}$ is Ricci-flat. We call the real metric $u_{ij} dx^i dx^j$ a semi-flat Calabi-Yau metric. Such metrics also may exist on manifolds which admit a flat affine connection ∇ on the tangent bundle and a volume form ω so that $\nabla\omega = 0$.

Recently, such metrics have been of interest in understanding mirror symmetry, and Gross-Wilson have constructed many such metrics on S^2 which are singular at 24 points, as real slices of limits of Calabi-Yau metrics on elliptic K3 surfaces. We construct many such metrics on S^2, singular at any $n \geq 6$ points, and compute the local affine structure near the singularities. The techniques involve affine differential geometry and a solving a semilinear PDE on S^2 minus singularities.