

**Mini-courses**  
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*Dynamics and Geometry of interval exchange  
maps and translation surfaces*

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**Abstract**

An interval exchange map (iem) is a one-to-one map of the interval which is locally a translation except at finitely many discontinuities; the simplest example of such maps are rotations on a circle (cut at some point to get an interval). To analyze the dynamics of such transformations, Rauzy, Veech and Zorich have developed a renormalization algorithm which generalizes the classical continuous fraction for irrational numbers. In some ways, iems are similar to rotations; for instance, almost all iems are uniquely ergodic (Masur, Veech); on the other hand, almost all iems with nonrotational combinatorics are weakly mixing, so quite different in this respect from rotations. Taking the suspension of an iem one gets a translation surface; this remarkable geometric object can also be seen as a Riemann surface equipped with a holomorphic one form. The Teichmüller space and the moduli space for these surfaces are of special interest. In particular the moduli space is a complex finite-volume orbifold equipped with an action of  $SL(2, \mathbb{R})$ , the restriction to the diagonal subgroup giving rise to a flow called the Teichmüller flow.

The first lecture will give a general overview of the subject, aimed at a general scientific audience, stressing ideas rather than precise mathematical statements. The following three will deal with some of the recent developments of this very active subject.

**Lecture 2. Interval exchange maps**

**Lecture 3. Translation surfaces**

**Lecture 4. Moduli space and the Teichmüller flow**