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Automorphisms of free algebras of Schreier varieties

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A variety of linear algebras over a field is Schreier if any subalgebra of a free algebra of this variety is free in the same variety of algebras. The main examples of Schreier varieties of algebras are the variety of all algebras, the variety of all commutative algebras, the variety of all anti-commutative algebras, varieties of all Lie algebras and superalgebras, varieties of all restricted Lie algebras and superalgebras. In this talk we consider automorphisms of free algebras of Schreier varieties of algebras. The automorphism groups of these free algebras of finite rank are generated by elementary automorphisms. Recently U.U.Umirbaev gave a presentation of the automorphism groups of free algebras of Schreier varieties by generators and defining relations.

We consider automorphism orbits of elements of free algebras. We show that if two finite systems of a free algebra of a Schreier variety of algebras are stably equivalent, then they are equivalent. A system of elements of a free algebra is said to be primitive if it is a subset of some set of free generators of this free algebra. Primitive elements distinguish automorphisms : endomorphisms sending primitive elements to primitive elements are automorphisms. Moreover, if some endomorphism preserves the automorphic orbit of a nonzero element, then it is an automorphism. Using free differential calculus we present an algorithm to find the rank of a system of elements and matrix criteria for a system of elements to be primitive. Based on these results we obtain fast algorithms to recognize primitive systems of elements of free algebras of Schreier varieties. Algorithms to construct complements of primitive systems of elements with respect to free generating sets are constructed and implemented. We consider isomorphisms of free algebras with one defining relation.

We discuss also automorphisms of polynomial algebras, free associative algebras, Leibniz algebras, free Poisson algebras.