Derivatives of the spectral function and Sobolev norms of eigenfunctions on a closed Riemannian manifold

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Abstract

Let \( e(x, y, \lambda) \) be the spectral function and \( \chi_\lambda \) the unit spectral projection operator, with respect to the Laplace–Beltrami operator on a closed Riemannian manifold \( M \). We generalize the one-term asymptotic expansion of \( e(x, x, \lambda) \) by Hörmander to that of \( \partial_x^\alpha \partial_y^\beta e(x, y, \lambda)|_{x=y} \) for any multi-indices \( \alpha, \beta \) in a sufficiently small geodesic normal coordinate chart of \( M \). Moreover, we extend the sharp \( L^2, L^p \) \( (2 \leq p \leq \infty) \) estimates of \( \chi_\lambda \) by Sogge to the sharp \( L^2, \text{Sobolev } L^p \) estimates of \( \chi_\lambda \). Finally we should mention that the wave equation method is essentially used in the proof.