

# Derivatives of the spectral function and Sobolev norms of eigenfunctions on a closed Riemannian manifold

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## Abstract

Let  $e(x, y, \lambda)$  be the spectral function and  $\chi_\lambda$  the unit spectral projection operator, with respect to the Laplace–Beltrami operator on a closed Riemannian manifold  $M$ . We generalize the one-term asymptotic expansion of  $e(x, x, \lambda)$  by Hörmander to that of  $\partial_x^\alpha \partial_y^\beta e(x, y, \lambda)|_{x=y}$  for any multi-indices  $\alpha, \beta$  in a sufficiently small geodesic normal coordinate chart of  $M$ . Moreover, we extend the sharp  $L^2, L^p$  ( $2 \leq p \leq \infty$ ) estimates of  $\chi_\lambda$  by Sogge to the sharp  $(L^2, \text{Sobolev } L^p)$  estimates of  $\chi_\lambda$ . Finally we should mention that the wave equation method is essentially used in the proof.