The general 1D Schrödinger equation as an exactly solvable problem

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Abstract

We review an exact WKB resolution method for general stationary Schrödinger equations with polynomial potentials in the complex plane. First, the explicit Wronskian of two particular solutions is translated into a bilinear functional equation linking the Dirichlet and Neumann zeta-regularized spectral determinants on a half-line—for the harmonic oscillator this reduces to the Gamma-function reflection formula, from which the exact spectrum follows at once. Recently, the general functional equation has itself been resolved into exact Bohr–Sommerfeld eigenvalue formulae, explicit in a broader sense: as fixed-point conditions, which nevertheless appear solvable by iteration, and which also relate to some Bethe–Ansatz equations for 2D statistical models. Similar but parametric (position-dependent) fixed-point conditions specify the Schrödinger solutions themselves, e.g., the eigenfunctions.