

8th International Conference on Symmetries and Integrability of Difference  
Equations (SIDE8) **June 22–28, 2008**  
8<sup>e</sup> Conférence internationale “Symétrie et intégrabilité des équations aux  
différences” (SIDE8) **22–28 juin, 2008**

## Umbral harmonic oscillator and its symmetries

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### Abstract

The umbral calculus is an old mathematical tool that began its first steps in the XVII century [1]. Since the second half of XIX [2, 3] it was applied in a systematic way although until the second half of XX [4, 5] there was not a formal theory. Recently it has been used to provide discrete representations of canonical commutation relations [6]–[12]. In this approach it can be used to map equations and solutions from a continuous frame to a discrete one. The theory works very well for polynomials, however this is not the case for its extension to non-polynomial functions. A complicated, and then interesting, problem in this formalism is the harmonic oscillator. Due to the non preservation of the complete set of symmetries after discretization, some divergences appear. To second physical solutions of the discrete harmonic oscillator we use a non standard form of the umbral calculus discretization. Some surprising results are obtained.

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