Homogeneous locally nilpotent derivations and embeddings of affine spaces

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Let \( m \) and \( n \) be positive integers such that \( n \geq m \) and let \( B \) be a polynomial ring in \( m + n + 1 \) variables over a field \( k \) of characteristic 0. We give a bijective correspondence between the equivalence classes of embeddings \( \mathbb{A}^m \to \mathbb{A}^n \) and the equivalence classes of sequences of mutually commuting locally nilpotent derivations \( \delta_i \) (\( 1 \leq i \leq m \)) on \( B \) in some form, which are homogeneous with respect to a \( \mathbb{Z} \)-grading on \( B \) and have slices. The intersection \( A \) of the kernels of \( \delta_i \) for \( 1 \leq i \leq m \) inherits the \( \mathbb{Z} \)-grading on \( B \). We show that \( A \) is a polynomial ring with homogeneous coordinates if and only if the corresponding embedding is rectifiable.