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Embeddings of sffine space using Chebyshev polynomials and their successors

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Part I.

Key algebraic properties of the Chebyshev polynomials $T_i(x) \in \mathbb{Z}[x]$ are established, followed by a discussion of the curves $T_i(x) = T_j(y)$ over any field of characteristic 0, where $\gcd(i,j) = 1$. These are polynomial versions of the classical Lissajous curves. Then, mappings $\mathbb{A}^1 \to \mathbb{A}^3$ defined by $\big(T_i(t), T_j(t), T_k(t)\big)$ are explored. In particular, it is determined which of these are embeddings. Moreover, for fixed i, j with $\gcd(i,j) = 1$, it is shown that there are at most a finite number of algebraically non-equivalent embeddings of the form (T_i, T_j, T_k). For the knot enthusiast, when the ground field is real, this procedure supplies a vast array of knots parametrized by polynomials. A table of knot types associated with low-degree embeddings is given.

Part II.

It is natural to ask : what are the analogues of Chebyshev polynomials in two or more variables ? Candidates $F_n(x,y) \in \mathbb{Z}[x,y]$ are defined and their algebraic content is discussed. These are used to define other affine space embeddings, including $\mathbb{A}^2 \to \mathbb{A}^5$.