

« GÉOMÉTRIE ALGÈBRIQUE AFFINE. UN ATELIER EN L'HONNEUR DE PETER RUSSELL »  
1–5 JUIN 2009

“AFFINE ALGEBRAIC GEOMETRY. A CONFERENCE IN HONOUR OF PETER RUSSELL”  
JUNE 1–05, 2009

## Uniqueness of $\mathbf{C}^*$ -action

HUBERT FLENNER

Department of Mathematics  
Ruhr University Bochum  
Universitaetsstrasse  
Bochum, NRW 44780  
GERMANY

hubert.flenner@rub.de

---

We give a classification of effective  $\mathbf{C}^*$ -actions on smooth affine surfaces up to conjugation in the full automorphism group and up to inversion  $\lambda \mapsto \lambda^{-1}$  of  $\mathbf{C}^*$ . If a smooth affine surface  $V$  admits more than one  $\mathbf{C}^*$ -action then it is known to be Gizatullin i.e., it can be completed by a linear chain of smooth rational curves. Our main result is :

Theorem. Let  $V$  be a smooth affine  $\mathbf{C}^*$ -surface. Then its  $\mathbf{C}^*$ -action is unique up to conjugation and inversion if and only if  $V$  does not belong to one of the following classes.

- (a)  $V$  is a toric surface ;
- (b)  $V = V(n)$  is a Danilov-Gizatullin surface of index  $n \geq 4$  ;
- (c)  $V$  is a special smooth Gizatullin surface of type I or II.

Furthermore,  $V$  admits at most two conjugacy classes of  $\mathbf{A}^1$ -fibrations  $V \rightarrow \mathbf{A}^1$  if and only if  $V$  is not one of the surfaces in (a) or (b). In our talk we describe the exceptions and some elements of the proof of this result.

*This is a joint work with S. Kaliman and M. Zaidenberg.*