Spherical rank rigidity and Blaschke manifolds

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Abstract

Let M^n be a complete Riemannian manifold whose sectional curvature is bounded above by 1. We say that M has positive spherical rank if along every geodesic there is a conjugate point at $t = \pi$ (geodesics are assumed to be parametrized by arc-length). In case M has positive spherical rank, the equality discussion of the Rauch Comparison theorem implies that along every geodesic of M we have a spherical Jacobi field i.e., a Jacobi field of the form $J(t) = \sin(t)E(t)$, where $E(t)$ is a parallel vector field along the geodesic.

This notion of rank is analogous to the notion of geometric rank for upper curvature bound 0 or upper curvature bound -1 studied by Ballmann, Burns, Spatzier, Eberlein, Hamenstaedt etc. In the case of spherical rank we show: Let M^n be a complete, simply connected Riemannian manifold with upper curvature bound 1 and positive spherical rank. Then M^n is isometric to a compact rank one symmetric space i.e., isometric to a sphere or projective space. This is joint work with Ralf Spatzier and Burkhard Wilking.