Stable bundles on Hopf manifolds

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Abstract

A Hopf manifold is defined as the quotient of the punctured n-space $\mathbb{C}^n \backslash \{0\}$ by an infinite cyclic group, generated by a contraction of $(\mathbb{C}^n, 0)$. If the contraction is multiplication by a diagonal matrix, then the Hopf manifoldis called *diagonal*. All Hopf manifolds are non-algebraic. In particular, every diagonal Hopf manifold is diffeomorphic to $S^1 \times S^{2n+1}$ and is thus non-Kählerian. A generic diagonal Hopf manifold possesses very few (holomorphic) curves, the only ones being n elliptic curves corresponding to the coordinate axes in $\mathbb{C}^n$. But there exist, nevertheless, Hopf manifolds with infinite families of curves. For example, if the contraction defining the manifold is a multiple of the identity, then it admits an elliptic fibration (without a section). In this talk, we describe stable holomorphic vector bundles on diagonal Hopf manifolds. In the case of elliptically fibred Hopf manifolds, one can exploit the elliptic fibration to obtain a classification these bundles via twisted Mukai-transforms. For generic Hopf manifolds such a construction is not possible; we study the geometry of these bundles by analysing their structure on neighbourhoods of the isolated elliptic curves.