

# Low-dimensional geometry—a variational approach

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## Abstract

What is special about 6,7 and 8 dimensions? Why do we study Calabi-Yau threefolds,  $G_2$  and  $\text{Spin}(7)$  manifolds? One approach is to look at the geometry of open orbits of Lie groups. There are very few of these but some of the open orbits of the general linear group lead directly into the geometry of manifolds in the above dimensions and in particular the geometrical structure on their moduli spaces. The split orthogonal groups  $\text{SO}(n, n)$  also appear in the lists through their spin representations and this provides an entrance into the world of generalized geometry, based on the bundle  $T \oplus T^*$ , which specializes to the study of pairs of metric connections with skew torsion, itself of some interest in string theory.

The four Lectures will be organized as follows:

1. Open orbits and stable forms.
2. Geometrical structures on moduli spaces.
3. Generalized Calabi-Yau structures.
4. Generalized  $G_2$  and  $\text{Spin}(7)$  structures.
5. Constructions