Potentially cristalline representations and associated p-adic representations of GL_2

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Abstract

Given a p-adic de Rham representation V of G_{Q_p}, one constructs from its Hodge–Tate weights and the Weil group representation on the Dieudonne module $D(V)$ a locally algebraic representation $\mathrm{Alg}(V)$ of $\mathrm{GL}_n(Q_p)$ $\big(n = \dim(V)\big)$. There should then be correspondance between the possible Hodge filtrations on $D(V)$ and commensurability classes of lattices in $\mathrm{Alg}(V)$. The completion of $\mathrm{Alg}(V)$ with respect to such a lattice gives a Banach space representation of $\mathrm{GL}_n(Q_p)$.

Here we are examing the case of certain potentially crystalline two-dimensional V, namely those which become crystalline over a tamely ramified extension of Q_p. The representation $\mathrm{Alg}(V)$ is then a tensor product with an algebraic representation and a smooth supercuspidal representation, which shows up in the de Rham cohomology of the first Drinfeld covering of the p-adic upper half plane. We explain how to relate Hodge filtrations on $D(V)$ with lattices in $\mathrm{Alg}(V)$ using the geometry of this covering.