Local-global compatibility in the p-adic Langlands programme

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Abstract

In this talk I will formulate a rather general local-global compatibility conjecture in the context of the p-adic Langlands programme for GL_2. The local objects are the unitary Banach space representations of $\mathrm{GL}_2(Q_p)$ conjecturally attached to two-dimensional p-adic representaitons of the local Galois group at p. The global object is the p-adically completed etale cohomology of the tower of modular curves—this is equipped with commuting actions of the global Galois group G_Q and of $\mathrm{GL}_2(A_f)$. (Here A_f is the ring of finite adeles.) The conjecture is as follows:

If V is an odd irreducible continuous 2-dimensional p-adic representation of G_Q, unramified away from finitely many primes, then the multiplicity space of V in the completed cohomology of modular curves (which is a $\mathrm{GL}_2(A_f)$ representation) is the tensor product of local representations, where at p, the local factor is the Banach space representation attached (at least conjecturally) to V locally at p, and at primes away from p, the local factor is given by the classical local Langlands correspondence.

For V that are potentially semi-stable at p, this conjecture is due to Breuil (and will be discussed in his talk). I will focus in my talk on motivating the conjecture in the case of more general representations V, and will describe some evidence for the conjecture in the case when V arises from a non-clasical point on the eigencurve. (For such V, the local correspondence has been constructed by Colmez.)