Abstract for “Families of Modular Forms and their Representations”

(For the colloquium B.M.)

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Ramanujan dealt with the arithmetic properties of the Fourier coefficients of some classical modular forms, and unearthed striking congruences that contain important number theoretic information. For example, fix an even, positive definite, unimodular lattice in 24-dimensional space; the number of vectors of given length in it is an arithmetic function difficult to understand, but nevertheless has a simple expression modulo 691. This same arithmetic function, now taken modulo 11, also has a simple expression once you know the arithmetic function mod 11 that counts the number of solutions modulo p, for all p, of the equation $y^2 + y = x^3 + x$. Ever since such discoveries, the search for *congruences* that unify otherwise seemingly remote problems has broadened to guide much number-theoretic research. For example, it played its role in the dramatic proof of modularity of elliptic curves over $\mathbf{Q}$, a decade ago. The standard way of organizing this web of congruences is to view modular forms, and the representations that are associated to them, as members of a continuous family (with p-adic parameters). This leads to one of the major themes of the workshop (to be) held Sept 12-16 in Montreal. The aim of this colloquium is to describe this theme to a nonspecialized audience.