

Generating all vertices of a polyhedron is hard

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Abstract

We show that generating all negative cycles of a weighted graph is a hard enumeration problem, in both directed and undirected cases. More precisely, given a family of negative cycles, it is NP-complete to decide whether this family can be extended or there is no other negative cycle in the graph.

As a corollary, we solve in the negative two well-known generating problems from linear programming:

(i) Given an (infeasible) system of linear inequalities, generating all minimal infeasible subsystems is hard. Yet, for generating maximal feasible subsystems the complexity remains open.

(ii) Given a (feasible) system of linear inequalities, generating all vertices of the corresponding polyhedron is hard. Yet, in case of bounded polyhedra the complexity remains open.

In 1979 Khachiyan proved that one vertex of a polyhedron can be found in polynomial time. In contrast, (ii) means that all vertices can not be efficiently generated, unless $P = NP$.

This is joint work with L. Khachiyan, E. Boros, K. Elbassioni, and K. Borys.