Polyhedral Computations
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Introduction to Polyhedral Computation

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Abstract

Polyhedral computation is a rapidly expanding field of mathematics which addresses the computational complexity of solving problems associated with convex polyhedra and search for efficient algorithms. One of the most fundamental problems is the *vertex enumeration problem* that is to list all vertices of a convex polytope given as the solution set to a system of m linear inequalities in d-variables. The problem in its dual form is the *facet enumeration problem* (or the *convex hull problem*) for a given set of m points in R^d. These problems of enumerative nature has the important characteristic: the size of output may be very large and cannot be polynomially bounded by the input size in general. Consequently the most natural question is to ask whether a problem can be solved in time polynomial in both the input size and the output size. We shall use the usual term *polynomially solvable* if this is the case. (This is consistent with the conventional use for the decision problems whose output is of constant length.)

In this talk, we review various fundamental problems in polyhedral computation. The main objectives are to understand the wealth of the problems investigated and to present both known results and open problems on polynomial solvability. The problems we discuss include the vertex enumeration, the arrangement/zonotope construction, the Minkowski addition of polytopes, the Gröbner fan construction and the parametric LP/LCP.