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Recent Progress in the Test Set Method for Integer Programming

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Abstract

A test set for a family of integer programs is a finite collection of integral vectors with the property that every feasible non-optimal solution of any integer program in the family can be improved by adding a vector in the test set. There has been considerable activity in the area of test sets and primal methods (e.g. Graver and Grøbner bases, the integral basis method, Scarf's neighbors of the origin, etc.), with applications in Optimization, Statistics and Combinatorics. In the past, test sets were considered problematic due to their large entry size or their difficulty of computation. Here we report on fresh progress, made in the past year, that yields interesting algorithmic results:

1) In joint work with Serkan Hosten Shmuel Onn and Qiang Wang. We created polynomial-time algorithm to canonically rewrite any polyhedral system $\{x\colon Ax = b, x \geq 0\}$ as a face of a $m \times n \times k$ axial transportation polytope. Axial transportation polytopes are very special. For instance, one can decide whether they are empty or not in only linear time without relying on linear programming algorithms. Using one explicitly known "light" Grobner basis for axial transportation problems we propose a new combinatorial algorithm for testing integer feasibility of polyhedra and report on experimental results and two heuristic variations.

2) We present a new kind of linear integer programs of variable dimension, but constructed from a fixed block matrix, that admits a polynomial time solution. Interestingly enough we employed in the proof algebraic techniques such as Graver test sets and the equivalence of augmentation and optimization oracles. We discuss several applications of our algorithm to multiway transportation problems and to packing problems. One important consequence of our results is a polynomial time algorithm for the d-dimensional integer transportation problem for long multiway tables. Another interesting application is a new algorithm for the classical cutting stock problem.

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This is joint work with R. Hemmecke, S. Onn, and R. Weismantel.