Orbital Convolutions, Wrapping Maps and e-Functions

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Abstract

The Kirillov character formula posits a relationship between coadjoint orbits and characters of irreducible representations of Lie groups. For compact groups, it can be understood as a kind of "dual" to the the homomorphism $\Phi(\mu *_{\mathfrak{g}} \nu) = \Phi(\mu) *_G \Phi(\nu)$, where μ and ν are Ad-invariant distributions of compact support on the Lie algebra of G, and a distribution μ on $\mathfrak{g}$ wraps to a distribution on G by the formula $\langle \Phi(\mu), \varphi \rangle = \langle \mu, j. \ \varphi \circ \exp \rangle$. Here j is the square root of the determinant of the exponential map. This formula allows for the "transfer" of central harmonic analysis on G to Ad-invariant Euclidean harmonic analysis on $\mathfrak{g}$. Interestingly, it is globally valid, in the sense that there is no restriction on the supports of the distributions (aside from compactness). For distributions of support $\{e\}$ it is the Duflo isomorphism.

Unfortunately, for groups which are not compact, there is a technical problem: the convolution of (necessarily noncompact) adjoint orbits is not defined. Recently, in joint work with Norm Wildberger, we have been able to get around this problem for motion groups of the form $V \rtimes K$ (V a vector space and K compact Lie), by use of invariant means and almost periodic functions. One can recover, in particular, Lipsman's character formula. I shall describe how this works.

Another extension of the wrapping formula is to compact symmetric spaces G/K, where central analysis is replaced by K-invariant analysis. Here, however, the convolution on the tangent space $\mathfrak{p}$ must be "twisted" by using an e-function in the sense of Rouvière. It turns out that these are also globally defined and an analogue of the wrapping formula holds, from which one can deduce character formulae etc.