WORKSHOP
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Diffusion of protein receptors on a cylindrical dendritic membrane with partially absorbing traps

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Abstract

We present a model of protein receptor trafficking within the membrane of a cylindrical dendrite containing small protrusions called spines. Spines are the locus of most excitatory synapses in the central nervous system and act as localized traps for receptors diffusing within the dendritic membrane. We treat the transverse intersection of a spine and dendrite as a spatially-extended, partially-absorbing boundary and use singular perturbation theory to analyze the steady-state distribution of receptors. We compare the singular perturbation solutions with numerical solutions of the full model and with solutions of a reduced one-dimensional model, and find good agreement between them all. We also derive a system of Fokker-Planck equations from our model and use it to exactly solve a mean first passage time (MFPT) problem for a single receptor traveling a fixed axial distance along the dendrite. This is then used to calculate an effective diffusion coefficient for receptors when spines are uniformly distributed along the length of the cable, and to show how a non-uniform distribution of spines gives rise to anomalous subdiffusion.