Network Design: Optimization and Algorithmic Game theory
August 14–16, 2006

# On the computational complexity of congestion games

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**Abstract**

Rosenthal's congestion games, network congestion games, and various extensions and special cases thereof have lately been the subject of intense interest, especially to determine their price of anarchy and stability. However, these notions only make sense if the existence of an equilibrium state is guaranteed. Even if that is the case, either concept might only be meaningful if a corresponding equilibrium can be efficiently computed.

We show that it is NP-hard to determine whether a weighted network congestion game has a pure-strategy Nash equilibrium. This is true regardless of whether flow is unsplittable or not. We also prove that the problem of deciding whether a bidirectional local-effect game has a pure-strategy Nash equilibrium is NP-complete, and that the problem of finding a pure-strategy Nash equilibrium in bidirectional local-effect games with linear local-effect functions (where the existence of a pure-strategy Nash equilibrium is guaranteed) is PLS-complete.

In addition, we study the complexity of finding globally optimal solutions to network congestion games. We consider several variants of the problem concerning the structure of the game and the properties of its associated cost functions. Though many of these variants are NP-hard, we identify some variants that are solvable in polynomial time.

If time permits, we will also discuss the price of anarchy and the price of stability for certain classes of local-effect games and for $k$-splittable network congestion games.

*Based on joint papers with Juliane Dunkel and Carol Meyers, respectively.*