Rational codes and free clopen submonoids of free profinite monoids

Benjamin Steinberg
bsteinbg@math.carleton.ca
School of Mathematics and Statistics
Carleton University
Herzberg Lab., 1125, Colonel By Drive
Ottawa, Ontario K1S 5B6
CANADA

Abstract

The free profinite monoid $\hat{A}^*$ is the completion of the free monoid A^* with respect to its profinite metric (the metric which says that two words are close if it takes a very large finite automaton to separate them). It is well known that clopen subgroups of free profinite groups are finitely generated free profinite groups. The situation for monoids is more complex.

Margolis, Sapir and Weil showed that the closed submonoid of $\hat{A}^*$ generated by a finite code $C \subseteq A^*$ is a free profinite monoid freely generated by C. However, purely topological considerations show that an infinite code cannot be a discrete basis for a free closed submonoid of $\hat{A}^*$. Nonetheless, we show that if C is a rational code, then the closed submonoid of $\hat{A}^*$ generated by C is a free profinite monoid topologically generated by the closure of C. Moreover, we show that every free clopen submonoid of $\hat{A}^*$ is of this form.

The proof techniques use classical ideas from the theory of codes such as: unambiguous automata, matrix representations and wreath products.

This is joint work with Jorge Almeida.