The Zeta Function of a Cyclic Language with Connections to Elliptic Curves and Chip-Firng Games

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Abstract

n their 1990 paper "Zeta Functions of Formal Languages", Jean Berstel and Christophe Reutenauer defined the Zeta Function of a Cyclic Language in an effort to tie together the Zeta Functions in algebraic geometry over finite fields and symbolic dynamics. With this spirit, we explore the Zeta Function of an elliptic curve E over a finite field F_q from a combinatorial angle, highlighting a connection between this Zeta Function and that of Languages. Along the way, we will obtain integral bivariate expressions for $N_k = \#E(F_{q^k})$, where F_{q^k} is a kth degree extension of finite field F_q, which have a combinatorial interpretation in terms of spanning trees of the wheel graph. We also will describe determinantal formulas for N_k, and a new sequence of polynomials, which we call elliptic cyclotomic polynomials. In addition, one of the important features of elliptic curves which makes them the focus of contemporary research is that they admit a group structure. During the remainder of this talk I will describe chip-firing games, how they provide a group structure on the set of spanning trees, and numerous ways that these groups are analogous to those of elliptic curves.

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