Recent Progress in Combinatorics on Words
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A **few** words about **skew** and **episkew** words

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Abstract

A finite or infinite word w over a 2-letter alphabet $\{a, b\}$ is said to be *balanced* if, for any two factors u, v of w of the same length, the number of a's (or equivalently b's) in each of u and v differs by at most 1. It is well-known that the balanced finite words are exactly the *finite Sturmian* (or *Christoffel*) words.

In the pioneering paper of Morse and Hedlund (1940), balanced infinite words are called 'Sturmian trajectories' and belong to three classes: aperiodic Sturmian; periodic Sturmian; and non-recurrent infinite words that are ultimately periodic (but not periodic), called *skew words*. That is, the family of balanced infinite words consists of the (recurrent) Sturmian words and the (non-recurrent) skew infinite words, all of whose (finite) factors are balanced.

In this talk, I will discuss a generalization of this concept to *episturmian words* – a natural extension of Sturmian words to an alphabet with more than two letters. Specifically, I will give a number of equivalent definitions of so-called *episkew* infinite words on a finite alphabet $\mathcal{A}$. Such infinite words are precisely the skew words when $|\mathcal{A}| = 2$. Further, I will show that the set of all infinite words whose factors are *finite episturmian* (i.e., *finite Arnoux-Rauzy words*) consists of the (recurrent) episturmian words and the (non-recurrent) episkew words.

Episkew words recently appeared in joint work with Jacques Justin and Giuseppe Pirillo. I will finish by briefly surveying some of our main results involving inequalities that characterize finite and infinite episturmian words in a *wide sense* (episturmian and episkew infinite words), which include the balanced words.