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On the critical exponent of generalized Thue-Morse sequences

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Abstract

In this talk, I will discuss recent work concerning the *critical exponent* (i.e., the largest fractional power) occurring in generalized Thue-Morse sequences of the form $\mathbf{t}_{b,m} := (s_b(n) \bmod m)_{n \geq 0}$, where $m \geq 1$, $b \geq 2$ are integers, and $s_b(n)$ denotes the sum of the digits in the base b representation of $n \in \mathbb{N}$. These sequences were studied in 2000 by J.-P. Allouche and J. Shallit, who proved that $\mathbf{t}_{b,m}$ is *overlap-free* if and only if $b \leq m$, in which case the critical exponent is 2. For $b > m$, I will show that the critical exponent $e_{b,m}$ of $\mathbf{t}_{b,m}$ is equal to $2b/m$ if m does not divide $b-1$, otherwise $e_{b,m}$ is equal to ∞ since $\mathbf{t}_{b,m}$ is purely periodic if m divides $b-1$. In the non-periodic case, I will also show that the critical exponent of the prefixes of $\mathbf{t}_{b,m}$ is b/m when $b > m$, and explicitly determine the factors in $\mathbf{t}_{b,m}$ having critical exponent $e_{b,m}$.

This is joint work with Srečko Brlek, Amy Glen, and Sébastien Labbé.