Sturmian fixed points of morphisms

Valérie Berthé
berthe@lirmm.fr
LIRMM
Université Montpellier II
161, rue Ada
34 392 Montpellier Cedex 5
FRANCE

Abstract

A Sturmian word $s_{\alpha,\rho}$ is a coding over a two-letter alphabet of the orbit of the point ρ of the one-dimensional torus under the action of the irrational rotation $R_\alpha : \; x \mapsto x + \alpha \pmod 1$. We first sketch a simple geometric proof of Yasutomi's characterization of all the pairs (α, ρ) such that the Sturmian word $s_{\alpha,\rho}$ is a fixed point of some primitive substitution. We also give an explicit expression of the corresponding substitutions, based on a joint work with H. Ei and H. Rao. We then focus on a notion of duality acting on Christoffel words. This involution extends to Sturmian morphisms, in the sense that it preserves conjugacy classes of these morphisms, which are in bijection with Christoffel words. Furthermore, it is the restriction of some conjugation of the automorphisms of the free group.

These latter results were obtained in collaboration with A. de Luca and C. Reutenauer.