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Defects of fixed points of some substitutions

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Abstract

Droubay, Justin, and Pirillo have shown in [1] that every finite word w contains at most $|w|+1$ different palindromes (the empty word being considered as a palindrome, too), where $|w|$ denotes the length of w. The difference between $|w|+1$ and the number of palindromes in the word w is called *defect* of w. We call a finite word w having the defect equal to zero, or, in other words, containing the maximal possible number $|w|+1$ of palindromes, *full*. We say that an infinite word is *full* if all of its prefixes are full. The following definition has been introduced in [1].

Definition 1. A finite word w satisfies the property Ju if there exists a palindromic suffix of w which occurs exactly once in w.

The definition can be reformulated in the following way: w verifies the property Ju if and only if the longest palindromic suffix of w occurs exactly once in w.

Proposition 1 ([1])**.** *Let w be a finite word. Then w is full if and only if each prefix $\hat{w}$ of w satisfies the property Ju, i.e., the longest palindromic suffix of $\hat{w}$ occurs only once in $\hat{w}$.*

Obviously, an infinite word can be full only if its language contains infinitely many palindromes. The authors of [1] have proved that sturmian and episturmian words are full.

In this talk, we will prove fullness of fixed points of some well-known substitutions, for instance, the substitution φ_β coding the distances between neighboring β-integers, the Thue-Morse substitution, the period-doubling substitution, the Rote substitution. The most important role will be played in all cases by description of the relation

between palindromes and their images. We will as well ask the question: Are all the fixed points of primitive substitutions, which contain infinitely many palindromes, full?

This is joint work with Edita Pelantova and Petr Ambroz (Czech Technical University and Doppler Institut).

References

[1] X. Droubay, J. Justin, G. Pirillo, *Episturmian words and some constructions of de Luca and Rauzy*, Theoret. Comput. Sci. **255** (2001), 539–553