The Local Statistical Distribution of the Zeros of Random Paraorthogonal Polynomials on the Unit Circle

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Abstract

We consider random paraorthogonal polynomials on the unit circle $\{\Phi_n\}_{n\geq 0}$ defined by random i.i.d. Verblunsky coefficients α_0, α_1, $\ldots, \alpha_{n-2}$ uniformly distributed in a disk of radius $r < 1$ and α_{n-1} uniformly distributed on the unit circle. For any n, the zeros of Φ_n are n random points on the unit circle.

We prove that for any $e^{i\theta} \in \partial\mathbb{D}$, the distribution of the zeros of Φ_n in intervals of size $O(\frac{1}{n})$ near $e^{i\theta}$ is the same as the distribution of n independent random points uniformly distributed on the unit circle (i.e., Poisson). This means that, for large n, there is no local correlation between the zeros of the considered random paraorthogonal polynomials.

This work parallels the celebrated results obtained by S. Molchanov on the local structure of the spectrum of random one-dimensional Schrödinger operators.