Fermi-Dirac Generators and Tests for Randomness

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Abstract

A Fermi-Dirac experiment $\mathrm{FDO}(n, N)$ is an ordered random sampling of size n without replacement from the integers from 1 to N. A complete sampling $\mathrm{FDO}(N, N)$ yields a string (random permutation of size N) that can be used to simulate Fermi-Dirac experiments for any n less than or equal to N. We refer to such strings, regardless of their origin, as Ordered Fermi-Dirac Generators, (FDO) generators. Because of their lack of collisions, such generators cannot hope to simulate Maxwell-Boltzmann (MB) experiments (in a previous publication, the authors showed this to still be the case even if truncation is used to overcome the lack of any collisions). All pseudo-random number generators (PRNG) are, at some level, FDO generators. They are routinely combined with some sort of projection to provide simulations of MB generators.

Another approach is possible. Instead of trying to produce a good MB PRNG directly, one can focus on producing a high quality PRNG that does a good job of simulating a complete FDO experiment, where the absence of collisions (Pauli exclusion principle) is an intrinsic property. There is a natural duality between unordered $\mathrm{FD}(n, N)$ experiments and Bose-Einstein $\big(\mathrm{BE}(N - n, n + 1)\big)$ experiments (consisting of distributing $N-n$ indistinguishable balls to $n+1$ cells). We develop the asymptotics of occupation statistics for BE experiments which have many analogues to known results for MB experiments. However, some of the results are completely new with no analogue in the MB theory. We show that one of these has considerable ability to distinguish between the performances of a fairly large number of standard PRNGs. Other tests give an indication of the degree to which algorithmic chaos has destroyed local correlations inherent in dynamical systems. Finally,

we demonstrate how a high quality FDO generator can be transformed into an MB generator of essentially the same quality.

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