

# On the Number of Vertices With a Given Degree in a Galton-Watson Tree

Nariyuki Minami  
minami@sakura.cc.tsukuba.ac.jp  
*Institute of Mathematics*  
*University of Tsukuba*  
*Tsukuba*  
*Ibaraki 305-8571*  
*Japan*

## Abstract

Let  $\{\omega\}$  be the realization of the Galton-Watson tree with offspring distribution  $\Pi = \{p_n\}_{n=0}^{\infty}$ , for which we shall assume  $p_0 > 0$  and  $p_0 + p_1 < 1$ . Namely  $\{\omega\}$  is a random tree obtained as the trajectory of a Galton-Watson branching process. We denote by  $Z(\omega)$  the total number of vertices, the total progeny, of the tree  $\omega$ , by  $Y_k(\omega)$  the total number of vertices in  $\omega$  which have exactly  $k$  children ( $k \geq 0$ ), and by  $\mathcal{Y}_k(\omega)$  the number of vertices with at most  $k$  children. We shall first show that  $\mathcal{Y}_k$  can be viewed as the total progeny of another Galton-Watson tree which is hidden in  $\omega$ . We shall then proceed to discuss the asymptotic behavior, as  $n \rightarrow \infty$ , of the joint probability distribution  $P(Z = n; \{Y_k\}_k \in \bullet)$ , and show some related limit theorems.