Liouville Theorems and Spectral Edge Behavior for Periodic Operators on Abeliang Coverings of Compact Manifolds

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Abstract

The talk will describe relations between Liouville type theorems for solutions of a periodic elliptic equation (or a system) on an abelian cover of a compact Riemannian manifold and the structure of the dispersion relation for this equation at the edges of the spectrum. Here one says that the Liouville theorem holds if the space of solutions of any given polynomial growth is finite dimensional. The necessary and sufficient condition for a Liouville type theorem to hold is that the real Fermi surface of the elliptic operator consists of finitely many points (modulo the reciprocal lattice). Thus, such a theorem generically is expected to hold at the edges of the spectrum. The precise description of the spaces of polynomially growing solutions depends upon a 'homogenized' constant coefficient operator determined by the analytic structure of the dispersion relation. In most cases, simple explicit formulas are found for the dimensions of the spaces of polynomially growing solutions in terms of the dispersion curves. The role of the base of the covering (in particular its dimension) is rather limited, while the deck group is of the most importance.

The results are also established for overdetermined elliptic systems, which in particular leads to Liouville theorems for polynomially growing holomorphic functions on abelian coverings of compact analytic manifolds.

Analogous theorems hold for abelian coverings of compact combinatorial or quantum graphs.

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