Random Exponentials: Limit Laws for Sums and Extreme Values

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Abstract

We study the limit distribution of exponential sums

$$S_N(t) = \sum_{i=1}^{N} e^{tX_i} \quad \text{as} \quad t \to \infty, \qquad N \to \infty,$$

where (X_i) are i.i.d. random variables. This problem has a number of applications in probability and random media theory. Two cases are naturally distinguished:

(A) $\operatorname{ess\,sup} X < \infty$ $(= 0)$, and

(B) $\operatorname{ess\,sup} X = \infty$.

We consider Weibull/Fréchet tails of the form

$$\mathrm{P}\{X > x\} \approx \exp\{-c(\pm x)^{\pm\varrho}\}.$$

Here "+" and "−" correspond to cases (B) and (A), respectively, and "≈" is interpreted using (normalized) regular variation of $\ln \mathrm{P}\{X > x\}$. Asymptotic growth of N relative to t is "measured" on the exponential scale $\exp\{\lambda H_0(t)\}$, where the rate function $H_0(t)$ is a certain asymptotic version of $H(t) = \pm \ln \mathrm{E} e^{tX}$. Our results reveal a remarkable universality of the limit picture in both cases (A) and (B). Namely, we show that there exist two critical points, $0 < \lambda_1 < \lambda_2 < \infty$, below which LLN and CLT, respectively, break down. For $\lambda < \lambda_2$ the limits are described by a certain parametric family of stable laws with characteristic exponent $\alpha = \alpha(\varrho, \lambda) \in (0, 2)$ and skewness parameter $\beta \equiv 1$. We also obtain limit theorems for extreme values of the

sample $\{e^{tX_i}, i = 1, \ldots, N\}$. In particular, we show that the limit of the (scaled) empirical extremal measure is given by a Poisson random measure on $(0, \infty)$ with mean $\mu(x, \infty) = x^{-\alpha}$.

The talk is based on joint work with G. Ben Arous and S. Molchanov.