The Universality Classes in the Parabolic Anderson Model

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Abstract

We discuss the long-time behaviour of the parabolic Anderson model, the Cauchy problem for the heat equation with random potential on $\mathbb{Z}^d$. We consider general i.i.d. potentials and show that exactly *four* qualitatively different types of intermittent behaviour can occur. These four universality classes depend on the upper tail of the potential distribution:

(1) tails at ∞ that are thicker than the double-exponential tails,

(2) double-exponential tails at ∞ studied by Gärtner and Molchanov,

(3) a new class called *almost bounded potentials*, and

(4) potentials bounded from above studied by Biskup and König.

The new class (3), which contains both unbounded and bounded potentials, is studied in both the annealed and the quenched setting. We show that intermittency occurs on unboundedly increasing islands whose diameter is slowly varying in time. The characteristic variational formulas describing the optimal profiles of the potential and of the solution are solved explicitly by parabolas respectively Gaussian densities.