Combinatorial Problems Raised by Statistical Mechanics
February 19–23, 2007

Complete monotonicity for inverse powers of some combinatorially defined polynomial

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Abstract

If P is a univariate or multivariate polynomial with real coefficients and strictly positive constant term, and β is a positive real number, it is sometimes of interest to know whether $P^{-\beta}$ has all nonnegative Taylor coefficients. Problems of this type go back at least to a celebrated paper of Szegö (1933). In this talk I give a combinatorial interpretation of Szegö's result and then generalize it to a statement about complete monotonicity. I go on to give two sufficient conditions for complete monotonicity of inverse powers of polynomials: one applying to determinantal polynomials (including the spanning-tree polynomials of graphs and, more generally, the basis generating polynomials of regular or complex unimodular matroids), and the other applying to quadratic forms (including the basis generating polynomials of rank-2 matroids). Finally, I discuss the relation with the half-plane property, and mention some open questions.

This is joint work with Alex Scott.