

Combinatorial Problems Raised by Statistical Mechanics
February 19–23, 2007

Multiple partitions, paths and conformal field theories

Pierre Mathieu
pmathieu@phy.ulaval.ca
Département de Physique
Université Laval
Pavillon Alexandre-Vachon, local 2634
Sainte-Foy, Québec G1K 7P4
CANADA

Abstract

The Burge-Bressoud correspondence relates partitions with conditions $f_j + f_{j+1} \leq k - 1$, where f_j is the frequency of the part j in the partition, to lattice paths of height $\leq k - 1$. Here such paths are put in correspondence with sets of $k - 1$ ordered partitions $(n^{(1)}, n^{(2)}, \dots, n^{(k-1)})$, such that $n_\ell^{(j)} \geq n_{\ell+1}^{(j)} + 2j$ together with a simple bound on $n_{m_j}^{(j)}$, where m_j is the number of parts in $n^{(j)}$. This bijection entails an elementary and constructive proof of the Andrews multiple-sum enumerating partitions with frequency conditions. Analogous results pertaining to jagged partitions are presented. Problems of statistical mechanics associated to the corresponding paths are identified