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An unbiased pointing operator for unlabeled structures

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Abstract

A very fruitful approach to enumerate a class of labeled structures (e.g. unrooted trees) is to consider instead the associated rooted class. The rooted class is easier to count because the root gives a starting point for a recursive decomposition. Then the n-th counting coefficient of the unrooted class is equal to the n-th coefficient of the rooted class divided by n (a structure of size n gives rise to n rooted structures). In the unlabeled setting, the pointing approach does not adapt straightforwardly, because a structure of size n can give rise to less than n pointed structures (rooting at two vertices in symmetric position gives the same rooted object). In this talk, we introduce a pointing operator such that each unlabeled unrooted structure of size n gives rise to n unlabeled pointed structures. To this aim, we have to point not only a vertex but a symmetric cycle of vertices. Combining our pointing operator with Polya theory allows us to count several classes of unlabeled structures, recovering enumerative results that can be found using Otter's dissymmetry theorem. In addition, we obtain efficient random generators on such classes.