

Combinatorial Problems Raised by Statistical Mechanics  
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# Loop gas, alternating sign matrices, plane partitions and orbital varieties: proofs and conjectures

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## **Abstract**

Lattice loop gas models as well as associated quantum spin chains give rise to many combinatorial coincidences, that we will try to sort out. We will show in particular various conjectural or proved relations with alternating sign matrices on one hand, and plane partitions with symmetries on the other, using integrability. An even more surprising outcome is an application to the algebraic geometry of nilpotent matrix varieties.