Combinatorial Problems Raised by Statistical Mechanics
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Permutation tableaux and the partially asymmetric exclusion process

Sylvie Corteel
corteel@lri.fr
LRI, CNRS
Université de Paris-Sud
Bâtiment 490
91405 Orsay Cedex
FRANCE

Abstract

The partially asymmetric exclusion process (PASEP) is an important model from statistical mechanics which describes a system of interacting particles hopping left and right on a one-dimensional lattice of N sites. It has been observed that the (unique) stationary distribution of the PASEP has remarkable connections to combinatorics — see for example the papers of Derrida, and Duchi and Schaeffer. We prove using two methods that in fact the probability of being in a particular state of the PASEP can be viewed as a certain weight generating function for permutation tableaux of a fixed shape. (This result implies the previous combinatorial results.) The first proof relies on the matrix ansatz of Derrida *et al*, and hence does not give an intuitive explanation of why one should expect the steady state distribution of the PASEP to involve such nice combinatorics. The second proof is totally combinatorial.

We define a Markov chain — which we call the PT chain — on the set of permutation tableaux which projects to the PASEP in a very strong sense. Furthermore, via the bijection from permutation tableaux to permutations, the PT chain can also be viewed as a Markov chain on the symmetric group. Another nice feature of the PT chain is that it possesses a certain symmetry which extends the "particle-hole symmetry" of the PASEP. More specifically, this is a graph-automorphism on the state diagram of the PT chain which is an involution; this has a simple description in terms of permutations.

This is joint work with Lauren Williams (Harvard).