Combinatorial Problems Raised by Statistical Mechanics
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The field of average tile orientations in random tilings with holes

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Abstract

The study of random tilings of planar lattice regions goes back to the solution of the dimer model in the 1960's by Kasteleyn, Temperley and Fisher, but received new impetus in the early 1990's, and has since branched out in several directions in the work of Cohn, Kenyon, Okounkov, Sheffield, and others.

In this talk, we focus on the interaction of holes in random tilings, a subject inspired by Fisher and Stephenson's 1963 conjecture on the rotational invariance of the monomer-monomer correlation on the square lattice. In earlier work, we showed that the correlation of a finite number of holes on the triangular lattice is given asymptotically by a superposition principle closely paralleling the superposition principle for electrostatic energy.

We now take this analogy one step further, by showing that the discrete field determined by considering at each unit triangle the average orientation of the lozenge covering it converges, in the scaling limit, to the electrostatic field.

Our proof involves a variety of ingredients, including combinatorial arguments, Laplace's method for the asymptotics of integrals, Newton's divided difference operator, and hypergeometric function identities.