

Negative hard-squares

Mireille Bousquet-Mélou
bousquet@labri.fr
LaBRI
Université Bordeaux 1
345 cours de la Libération
Talence Cedex, Gironde 33405
FRANCE

Abstract

The independent sets of a graph $G = (V, E)$ are the subsets of V that contain no pairs of adjacent vertices.

Counting the independent sets of a rectangular grid is an open problem that is of interest to combinatorialists and physicists. In physics, this problem is usually known as the hard-square problem.

Two years ago, three physicists published some strange conjectures dealing with the “alternating” number of independent sets. That is, with the difference between the number of such sets with an even and odd cardinality. Some of these conjectures have been proved by Jakob Jonsson.

We will discuss these conjectures, and show that similar results hold, in greater generality, for other pieces of the square lattice. The latter results have been obtained in collaboration with Svante Linusson and Eran Nevo.