Combinatorial Problems Raised by Statistical Mechanics
February 19–23, 2007

Solution to some problems inspired by Mayer's theory of cluster integrals

Olivier Bernardi
olivier.bernardi@labri.fr
CRM
Centre de Recerca Matematica
Apartat 50
Bellaterra, Barcelona E-08193
ESPAGNE

Abstract

Mayer's theory of cluster integrals is a general algebraic transformation allowing one to write the partition function of a gas model as a generating function of weighted graphs. The choice of the interaction potential between particles in the gas determines the weights of the graphs. For instance, in the case of the one-dimensional hard-particle model, the weights $w(G)$ of a connected graph $G = (V, E)$ on n vertices is up to sign, the volume of a polytope Π_G:

$$w(G) = (-1)^{|E|}\mathrm{vol}(\Pi), \quad \text{where} \quad \Pi_G = \{(x_1, \ldots, x_n)/\ x_1 = 0 \text{ and } |x_i - x_j| < 1 \text{ for all edge } (i,j) \in E\}.$$

Using Mayer's transformation, Labelle, Leroux and Ducharme have established two expressions for the pressure in the hard-particle model and noticed that comparing these two expressions leads to the identity:

$$\sum_G w(G) = (-n)^{n-1},$$

where the sum is over all connected graphs on n vertices.

In this talk I will explain how to prove this identity by combinatorial arguments.