The G-orbit Hilbert schemes and McKay correspondence for simple singularities

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Abstract

For a given finite subgroup G of $\mathrm{SL}(2, \mathbf{C})$ there is a famous McKay correspondence between the minimal resolution of the quotient $\mathbf{C}^2/G$ and irreducible representations of G. The G-orbit Hilbert scheme of $\mathbf{C}^2$ is, by definition, the Hilbert scheme which parametrizes all the zero dimensional G-invariant subschemes (= G-clusters), each with structure sheaf isomorphic to the regular representation of G. This turns out to be a minimal resolution of $\mathbf{C}^2/G$. Thus any point of the minimal resolution of $\mathbf{C}^2/G$ has a nontrivial interpretation as a G-module, the defining ideal of the G-cluster. This enables us to give an explanation for the McKay correspondence, other than that given by Gonzalez-Springberg and Verdier. We report this and related results, based on *joint works with Ito, Shinoda and Gomi.*