Groups and symmetries: From the Neolithic Scots to John McKay
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Arithmetic of the root system E_8 and the geometry of the moduli spaces of $K3$ surfaces.

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Abstract

The global Torelli theorem for projective $K3$ surfaces was first proved by Piatetskii-Shapiro and Shafarevich 36 years ago, opening the way to treating moduli problems for $K3$ surfaces. The moduli space of polarised $K3$ surfaces of degree $2d$ is a quasi-projective variety of dimension 19. For general d very little has been known hitherto about the Kodaira dimension of these varieties. In this talk we present an almost complete solution to this problem. Our main result obtained by myself together with K. Hulek and G. Sankaran says that this moduli space is of general type for $d > 61$ and for $d = 46$, 50, 54, 57, 58, 60. In order to prove this theorem we solve a general problem on the finite quotient singularities of the modular varieties of orthogonal type. The Borcherds products and the arithmetic of the root lattice E_8 play also an important role in the proof.