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Families of Ramanujan graphs and quaternion algebras

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Abstract

Using quaternion algebras over a totally real field one can construct families of Ramanujan graphs as quotients of a Bruhat-Tits tree by a discrete subgroup coming from the quaternion algebra. Such constructions were made by Lubotzky-Phillips-Sarnak and Pizer for rational quaternion algebras and by Jordan-Livné for quaternion algebras over totally real fields. The cases we consider are special because they are connected to arithmetic geometry and theta series, and so our emphasis is different than Jordan-Livné's. We also consider nested families of Ramanujan graphs and some new questions arising in that context.

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