Groups and symmetries: From the Neolithic Scots to John McKay April 27 – 29, 2007

Painlev, Klein and the icosahedron

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Abstract

We will describe how finite groups (and a few infinite groups) help to find algebraic solutions of the Painlevé VI equation. This is a second order nonlinear ODE which should be viewed as the "nonlinear version" of the Gauss hypergeometric equation, as will be explained. Thus the thrust of the talk is aimed at finding the nonlinear analogue of Schwarz's famous list of algebraic solutions of the hypergeometric equation. In particular we will show that the resulting list is richer than the finite subgroups of SU(2). (Along the way we establish two other more obvious extensions of Schwarz's list to certain *nonrigid* linear ODEs—the main problem here being the construction of nonrigid ODEs with given finite monodromy groups.)