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Critical behavior in non-intersecting path ensembles

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Abstract

I will discuss new critical behavior in two ensembles of non-intersecting path. The models have in common that they are determinantal point processes that can be analyzed with multiple orthogonal polynomials and the associated Riemann–Hilbert problem. The size of the Riemann–Hilbert problem is either 4×4 or 3×3 and their asymptotic analysis presents certain novel features. The first model consists of n non-intersecting Brownian bridges with two starting and two ending positions. Depending on the separation between these positions there are three different regimes in the large n limit. In the regime of large separation the Brownian paths fill two disjoint ellipses, and in the regime of small separation the paths fill a simply connection region bounded by an algebraic curve of degree six. In the regime of critical separation, a surprising connection is found with the Hastings–McLeod solution of the Painlevé II equation.

This is joint work with Steven Delvaux (Leuven).

The second model is a model of non-intersecting squared Bessel paths that start at a positive value and end at 0. The model has a wall (hard edge) at 0 and the paths are forced to come to the wall since they are conditioned to end at 0. A new phase transition takes place when the paths first come to the wall.

This is joint work with Andrei Martinez Finkelshtein (Almeria) and Franck Wielonsky (Lille).