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Coupled random matrices and biorthogonal polynomials

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Abstract

I will report on some recent **joint work with Arno Kuijlaars** on the coupled random matrix model. We consider the eigenvalue statistics of two $n \times n$ Hermitian matrices, M_1 and M_2, taken randomly with respect to the probability measure

$$\frac{1}{Z_n} \mathrm{e}^{-n \operatorname{Tr}(V(M_1)+W(M_2)-\tau M_1 M_2)} \mathrm{d}M_1 \, \mathrm{d}M_2$$

Here V and W are two polynomials of even degree, $\tau \neq 0$ is called the coupling constant and Z_n is a normalization constant. The eigenvalue statistics of M_1 and M_2 can be expressed in terms of certain biorthogonal polynomials. These are two families of monic polynomials $p_{k,n}(x) = x^k + \cdots$ and $q_{j,n}(y) = y^j + \cdots$ satisfying the relation

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_{k,n}(x) q_{j,n}(y) \mathrm{e}^{-n(V(x)+W(y)-\tau xy)} \mathrm{d}x \, \mathrm{d}y = 0, \qquad \text{if } j \neq k.$$

If one has a complete asymptotic description of these polynomials then it is possible to compute the limiting behavior of the eigenvalue correlations. I will discuss a rigorous approach to obtain asymptotics for the polynomials $p_{n,n}$ and the limiting eigenvalue correlations of M_1 in the special case that $W(y) = y^4/4$ and V is an even polynomial. A particular feature in the analysis is the introduction of an equilibrium problem that characterizes the limiting zero distribution of the polynomials $p_{n,n}$. We use this equilibrium to apply the Deift/Zhou steepest descent analysis for a 4×4 Riemann–Hilbert problem associated to the polynomials $p_{n,n}$.