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Higher order analogues of the Tracy–Widom distribution and the Painlevé II hierarchy

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Abstract

The limiting distribution of the largest eigenvalue in unitary random matrix ensembles is generically described by the Tracy–Widom distribution, which can be expressed either as a Fredholm determinant of an Airy kernel operator, or explicitly in terms of the Hastings-McLeod solution to the Painlevé II equation. The Tracy–Widom distribution appears whenever the limiting mean eigenvalue density vanishes like a square root at the edge of the spectrum. Our focus is on critical ensembles where the limiting mean eigenvalue density vanishes faster than a square root near the edge. Here the limiting distribution of the largest eigenvalue is described by a Fredholm determinant of an operator related to the Painlevé I hierarchy. We show that this determinant can be expressed in terms of a special smooth solution to a higher order Painlevé II equation contained in the Painlevé II hierarchy. In addition we obtain large gap asymptotics for the determinant.

This is joint work with A. Its and I. Krasovsky.