Higher order analogues of the Tracy–Widom distribution and the Painlevé II hierarchy

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**Abstract**

The limiting distribution of the largest eigenvalue in unitary random matrix ensembles is generically described by the Tracy–Widom distribution, which can be expressed either as a Fredholm determinant of an Airy kernel operator, or explicitly in terms of the Hastings-McLeod solution to the Painlevé II equation. The Tracy–Widom distribution appears whenever the limiting mean eigenvalue density vanishes like a square root at the edge of the spectrum. Our focus is on critical ensembles where the limiting mean eigenvalue density vanishes faster than a square root near the edge. Here the limiting distribution of the largest eigenvalue is described by a Fredholm determinant of an operator related to the Painlevé I hierarchy. We show that this determinant can be expressed in terms of a special smooth solution to a higher order Painlevé II equation contained in the Painlevé II hierarchy. In addition we obtain large gap asymptotics for the determinant.

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