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## Asymptotics of Tracy–Widom distributions

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### **Abstract**

The Tracy–Widom distribution functions describe the distribution of a variety of statistical quantities, such as the largest eigenvalue of a matrix drawn from the Gaussian orthogonal, symplectic, or unitary ensembles (GOE, GSE, or GUE). By using the standard representations of the Tracy–Widom distributions as integrals of Painlevé functions starting from positive infinity it is straightforward to obtain the asymptotic expansion of the right-hand tail, whereas that of the left-hand tail contains a more elusive constant. Using the Riemann–Hilbert steepest descent method we obtain new representations for the Tracy–Widom distributions in terms of integrals of Painlevé functions starting from negative infinity. Using this we give the first proof of the GOE and GSE constants, as well as a second proof of the recently proven result for GUE. We conclude by discussing current work on the constant problem for the Tracy–Widom distributions for the largest observed eigenvalue, when each eigenvalue has a given probability of being observed.

**This is joint work with Jinho Baik and Jeffery DiFranco.**