Non-Hermitian RMT applied to QCD

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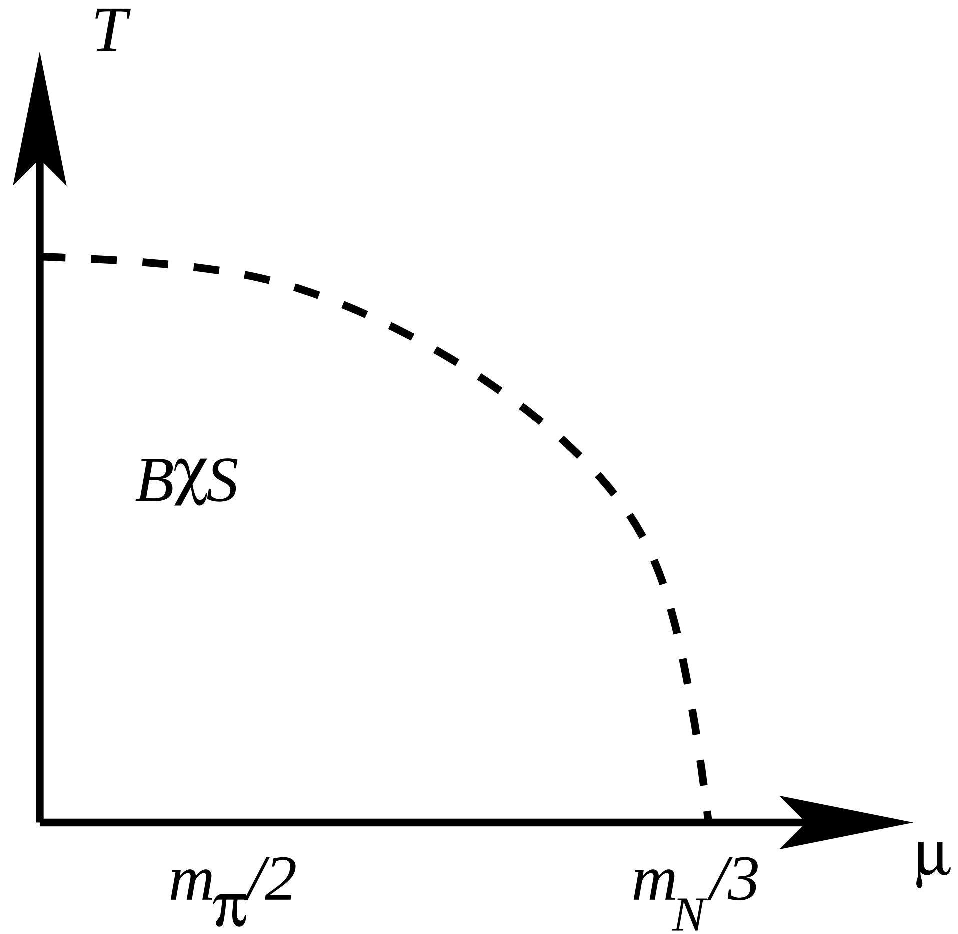
Setup RMT for QCD

Feature Chemical potential and Non-Hermiticity

Focus Chiral symmetry breaking

Key-words Orthogonal polynomials in the complex plane
Sign problem
Non-positive densities
Toda lattice equations
Cauchy transforms
Bosonic theories

The Big Picture



RMT for QCD

The QCD partition function

Path integral

$$Z_{QCD} = \int \mathrm{d}A \, \det(iD_\eta \gamma_\eta)^{N_f} \, e^{-S_{\mathrm{YM}}(A)}$$

The Dirac operator $iD_\eta \gamma_\eta$ will be the random matrix.

Properties of the QCD Dirac operator

Anti-Hermitian

$$(iD_\eta\gamma_\eta)^\dagger = -iD_\eta\gamma_\eta$$

Axial-Symmetry

$$\{iD_\eta\gamma_\eta, \gamma_5\} = 0$$

No additional symmetries

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$$(iD_\eta\gamma_\eta) = \left(\begin{array}{cc} 0 & iW \\ iW^\dagger & 0 \end{array} \right)$$

W has complex matrix elements.

The QCD partition function

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The chGUE partition function

$$\mathcal{Z} \equiv \int dW \, \det(\mathcal{D})^{N_f} \, e^{-N \mathrm{Tr} W^\dagger W}$$

where

$$\mathcal{D} = \begin{pmatrix} 0 & iW \\ iW^\dagger & 0 \end{pmatrix}$$

same flavor symmetries

Shuryak, Verbaarschot, NPA **560**, 306 (1993), Verbaarschot, PRL **72**, 2531 (1994)

Selvin, Nagao, PRL 70 (1993) 635, Zirnbauer, J. Math. Phys. 37 (1996) 4986

The quark mass m and chemical potential μ

$$Z_{QCD} = \int dA \, \det{}^{N_f} (iD_\eta \gamma_\eta + \mu\gamma_0 + m) \, e^{-S_{\mathrm{YM}}}$$

Anti Hermitian

Hermitian

The sign problem: *The measure is not real and positive*

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$$Z_{QCD} = \int dA \det{}^{N_f}(iD_\eta\gamma_\eta + \mu\gamma_0 + m)\, e^{-S_{\rm YM}}$$

Anti Hermitian Hermitian

The sign problem: *The measure is not real and positive*

A problem for Monte Carlo

chRMT by Stephanov

$$\mathcal{Z} \equiv \int dW \, \det(\mathcal{D}(m;\mu))^{N_f} \, e^{-\frac{N}{2}\mathrm{Tr}W^\dagger W}$$

where

$$\mathcal{D}(m;\mu) = \begin{pmatrix} m & iW+\mu \\ iW^\dagger+\mu & m \end{pmatrix}$$

Stephanov, PRL **76**, 4472 (1996)

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Same explicit symmetry breaking of flavor symmetries

Same hermiticity properties as in QCD

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No eigenvalue representation known

Stephanov, PRL **76**, 4472 (1996)

chRMT by Osborn

$$\mathcal{Z}_N^{N_f}(m;\mu) \quad \equiv \quad \int dW d\Psi \det{}^{N_f}(\,\mathcal{D}(\mu) + m\,)e^{-N\mathrm{Tr}W^\dagger W}e^{-N\mathrm{Tr}\Psi^\dagger \Psi}$$

where the *Dirac operator* is given by

$$\mathcal{D}(\mu) = \begin{pmatrix} 0 & iW + \mu\Psi \\ iW^\dagger + \mu\Psi^\dagger & 0 \end{pmatrix}$$

Osborn PRL 93 (2004) 222001

chRMT by Osborn

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Eigenvalue representation known

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chRMT by Osborn

Eigenvalue representation

$$\begin{aligned} \mathcal{Z}_N^{N_f}(m;\mu) &= \int \prod_{k=1}^{N} d^2 z_k \left|\Delta_N(\{z_l^2\})\right|^2 |z_k|^{2\nu+2} \\ &\times K_\nu\left(\frac{N(1+\mu^2)}{2\mu^2}|z_k|^2\right) e^{-\frac{N(1-\mu^2)}{4\mu^2}(z_k^2+{z_k^*}^2)}(m^2-z_k^2)^{N_f} \end{aligned}$$

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chRMT by Osborn

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The eigenvalue density from the orthogonal polynomials

$$\rho_N^{N_f=1} = 2w(z,z^*;\mu)\sum_{k=0}^{N-1}\frac{p_k(z^*)(p_k(z)-p_N(z)p_k(m)/p_N(m))}{r_k}$$

where

$$p_k(z;\mu) = \left(\frac{1-\mu^2}{N}\right)^k k! L_k^\nu\left(-\frac{Nz^2}{1-\mu^2}\right)$$

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G. Akemann Int.J.Mod.Phys. A22 (2007) 1077

chRMT by Osborn

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where Not real and positive

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Definition of the eigenvalue density

Eigenvalue equation

$$(iD_\eta \gamma_\eta + \mu\gamma_0)\psi_j = z_j \psi_j$$

Eigenvalue density

$$\boxed{\rho^{N_f}(z, z^*, m; \mu) \quad \equiv \quad \left\langle \sum_j \delta^2(z - z_j) \right\rangle_{\mathrm{QCD}}}$$

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$$\langle \mathcal{O} \rangle_{\mathrm{QCD}} \equiv \frac{\int \mathrm{d}A\, \mathcal{O} \det(iD_\eta\gamma_\eta + \mu\gamma_0 + m)^{N_f}\ \mathrm{e}^{-S_{\mathrm{YM}}(A)}}{\int \mathrm{d}A \det(iD_\eta\gamma_\eta + \mu\gamma_0 + m_f)^{N_f}\ \mathrm{e}^{-S_{\mathrm{YM}}(A)}}$$

Sign problem $\Rightarrow \rho$ complex valued

Sign problem $\Rightarrow \rho$ complex valued

What is this good for ?

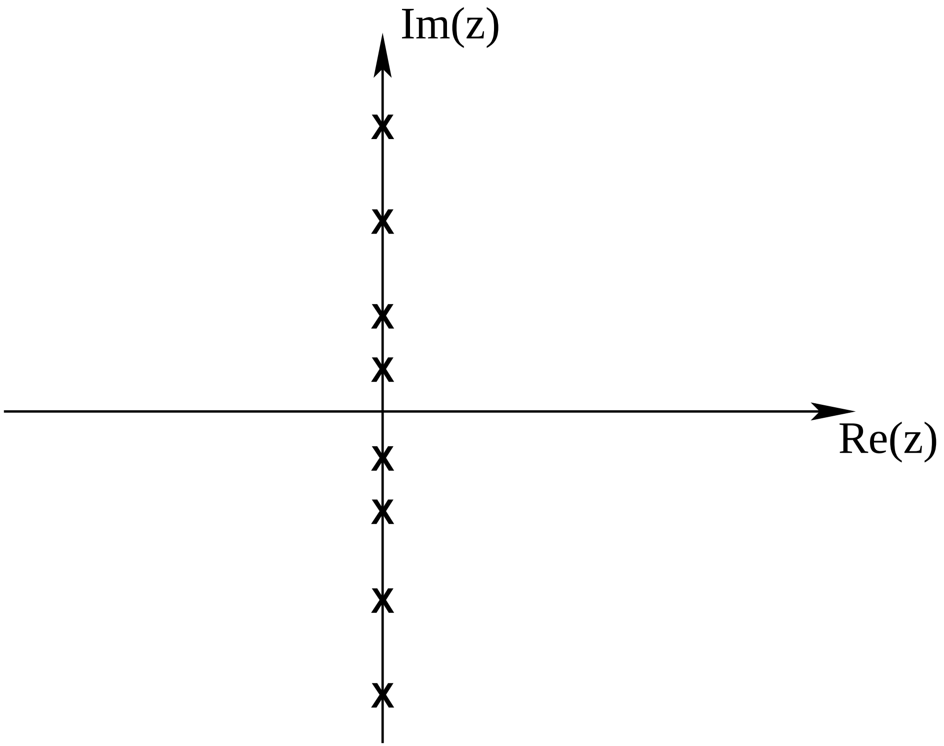
The Silver Blaze Problem

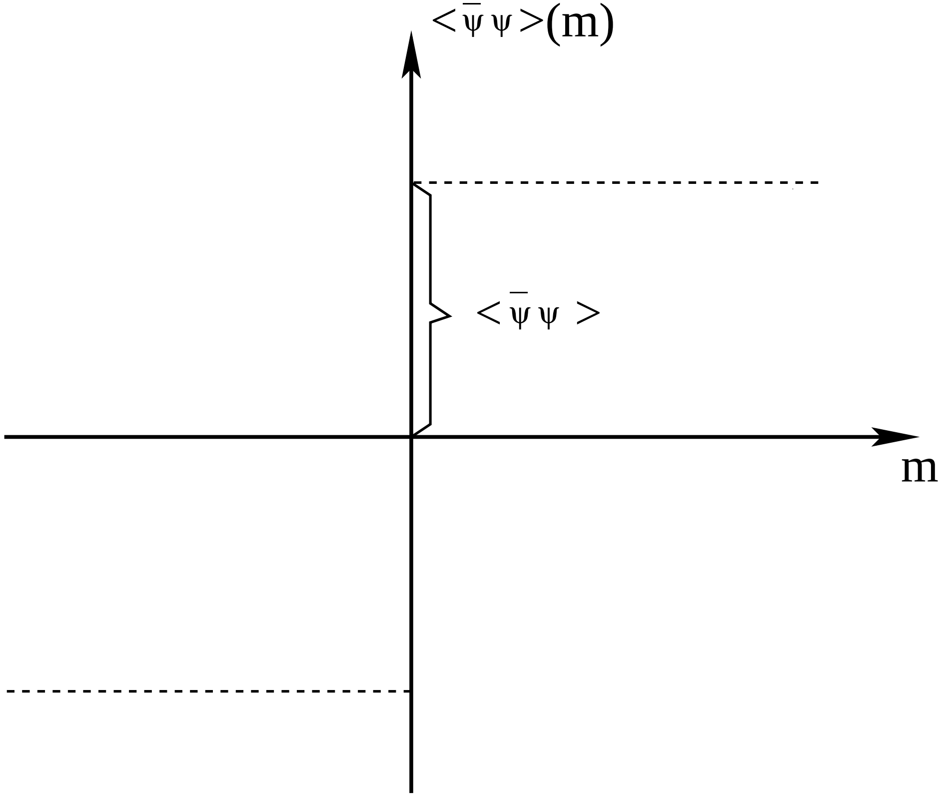
Sir Arthur Conan Doyle *The Memoirs of Sherlock Holmes: Silver Blaze*

Thomas D . Cohen PRL (2003) 222001

$\mu = 0$

Banks Casher

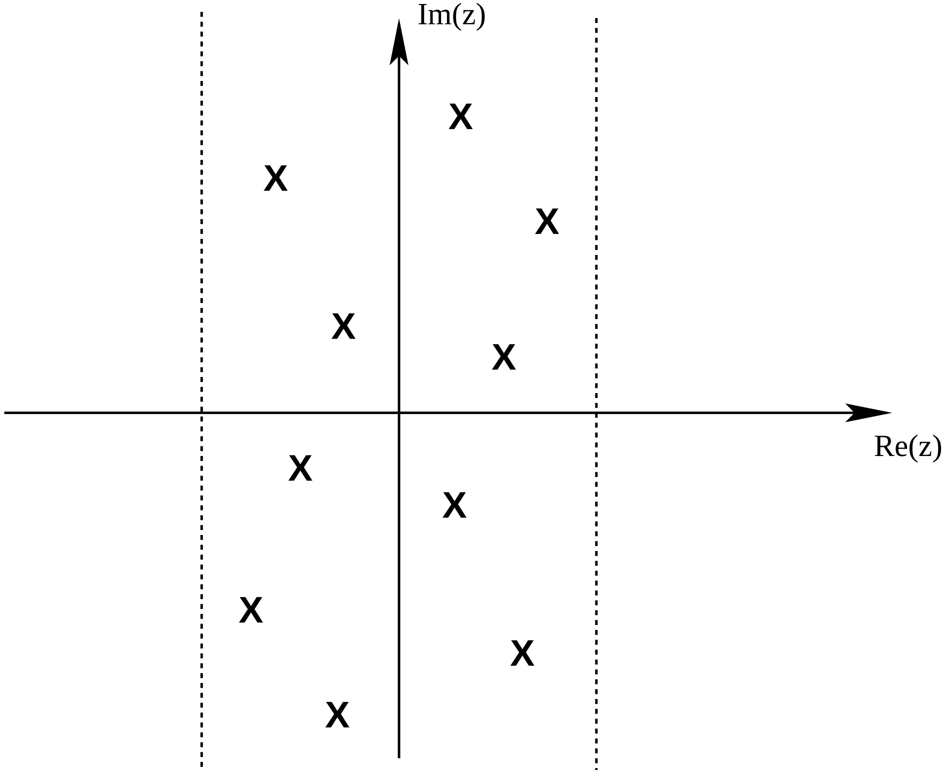


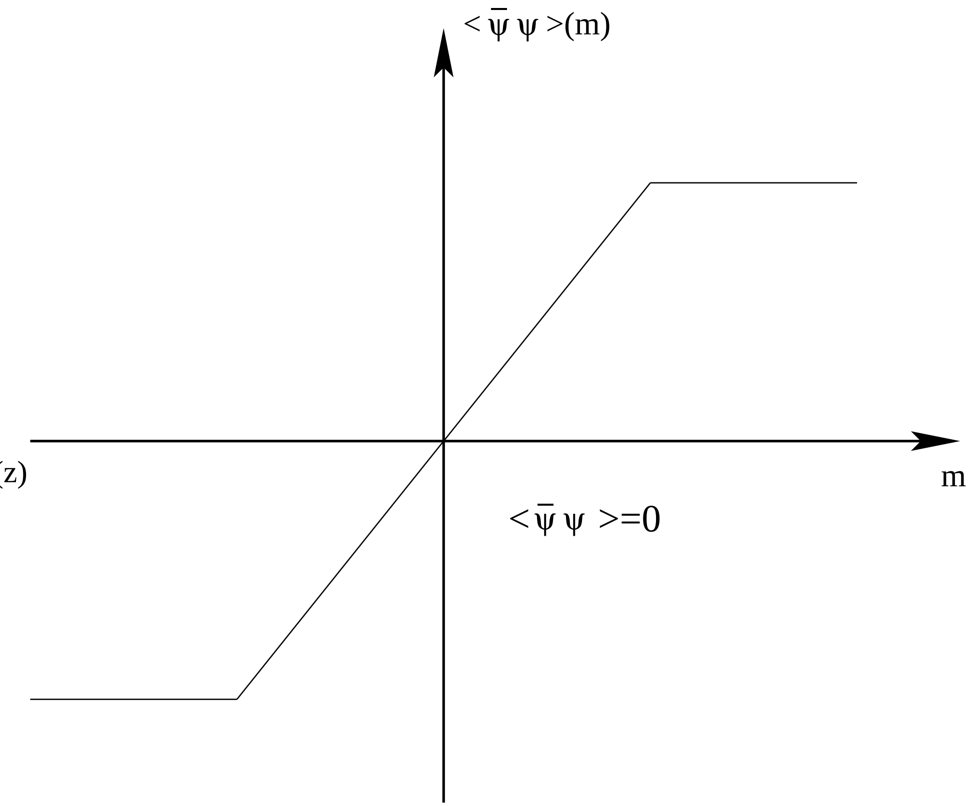


$$\boxed{\langle \bar{\psi}\psi \rangle = \frac{\pi}{V}\rho(0)}$$

Banks Casher NPB 169 (1980) 103

Electrostatic analogy suggests



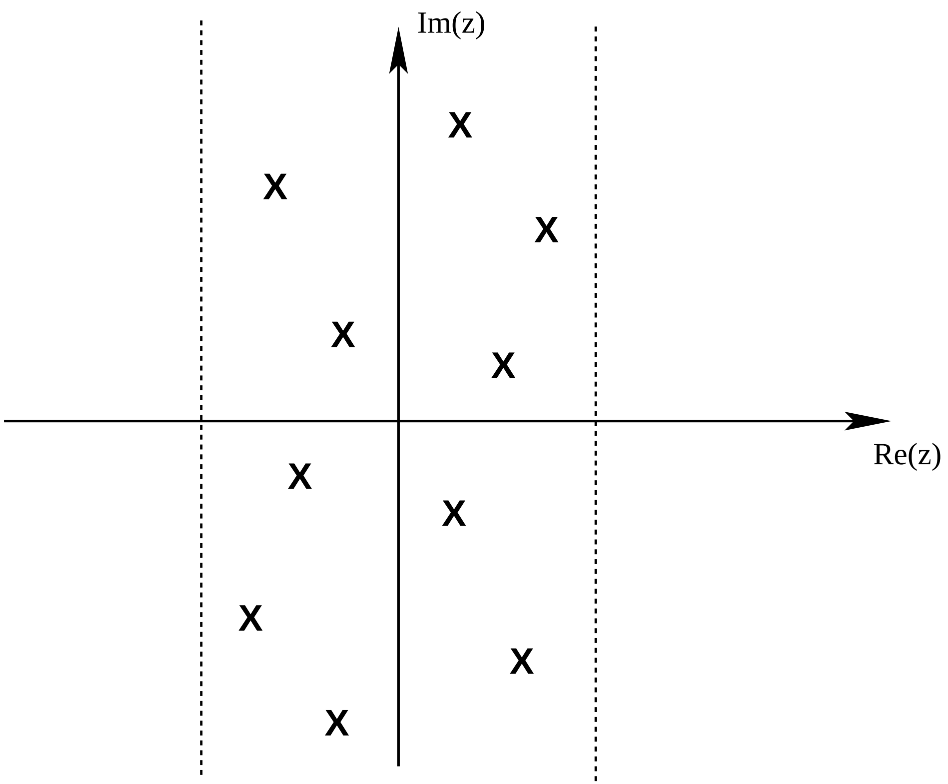


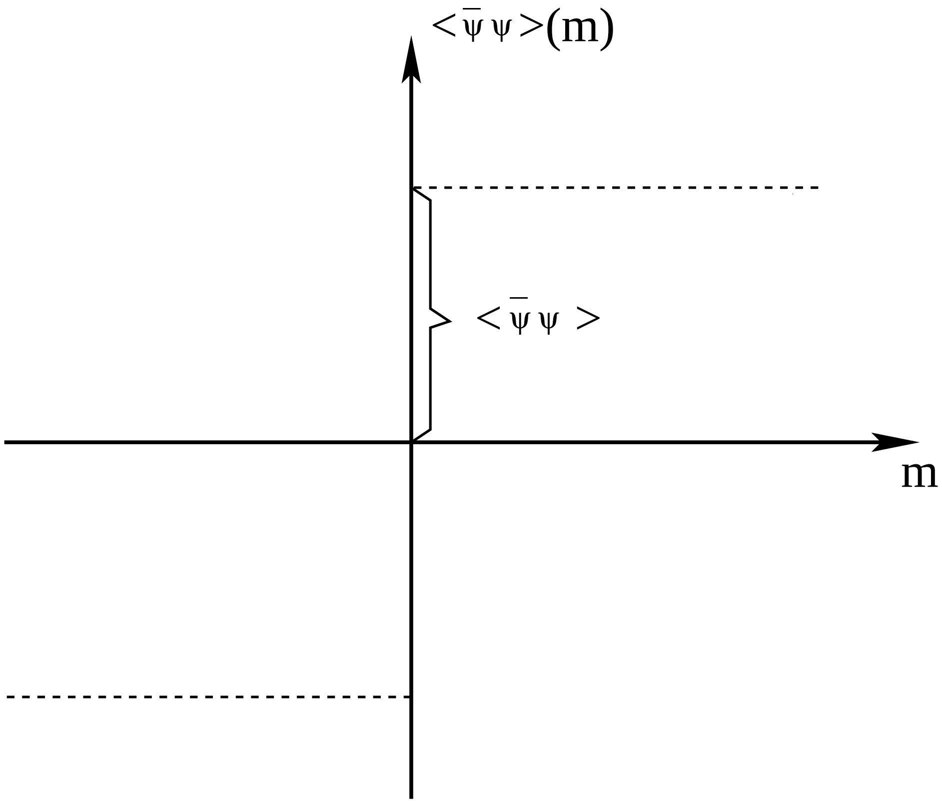
Electrostatic analogy:

Eigenvalues = charges, quark mass = test charge

Barbour et al. NPB 275 (1986) 296

$\mu \neq 0$ *The silver blaze problem*





Eigenvalues move into the complex plane

the discontinuity of the chiral condensate remains

Barbour et al. NPB 275 (1986) 296

Gibbs PLB 182 (1986) 369

Cohen PRL 91 (2003) 222001

We need Microscopic-regime of QCD

The eigenvalue density $z\langle\bar\psi\psi\rangle \ll \frac{1}{\sqrt{V}}$

SBχS The basic assumption

Chiral limit $m\langle\bar\psi\psi\rangle \ll \frac{1}{\sqrt{V}}$

Small chemical potential $\mu^2 F_\pi^2 \ll \frac{1}{\sqrt{V}}$

Notice $\mu \sim m_\pi$

Gasser, Leutwyler, PLB 184 (1987) 83, PLB 188 (1987) 477
Neuberger, PRL 60 (1988) 889
Leutwyler, Smilga, PRD 46 (1992) 5607
Shuryak, Verbaarschot, NPA 560 (1993) 306
Stephanov PRL 76 (1996) 4472
Akemann PRL 89 (2002) 072002, J.Phys. A36 (2003) 3363
Splittorff, Verbaarschot, NPB 683 (2004) 467
Osborn PRL 93 (2004) 222001
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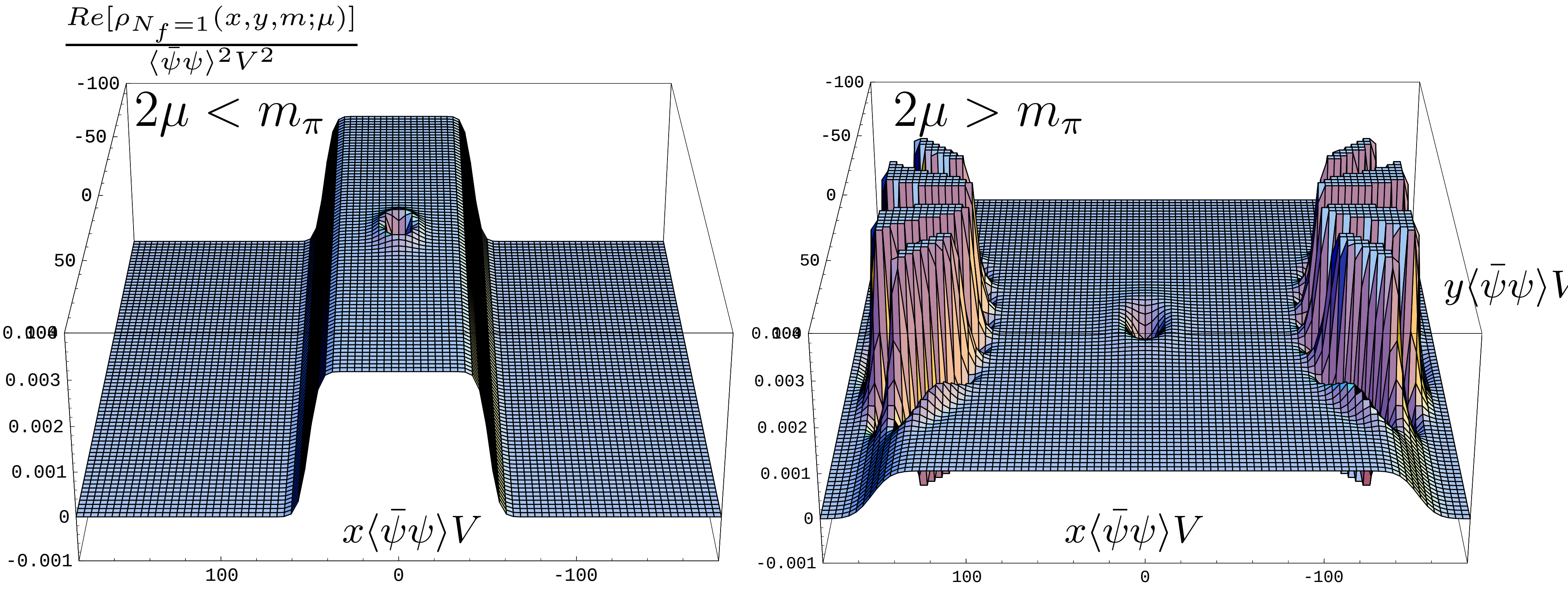
The unquenched eigenvalue density

$m\langle\bar{\psi}\psi\rangle V = 100$ increasing $2\mu^2 F_\pi^2 V$



For $2\mu > m_\pi$ the density is complex and oscillates

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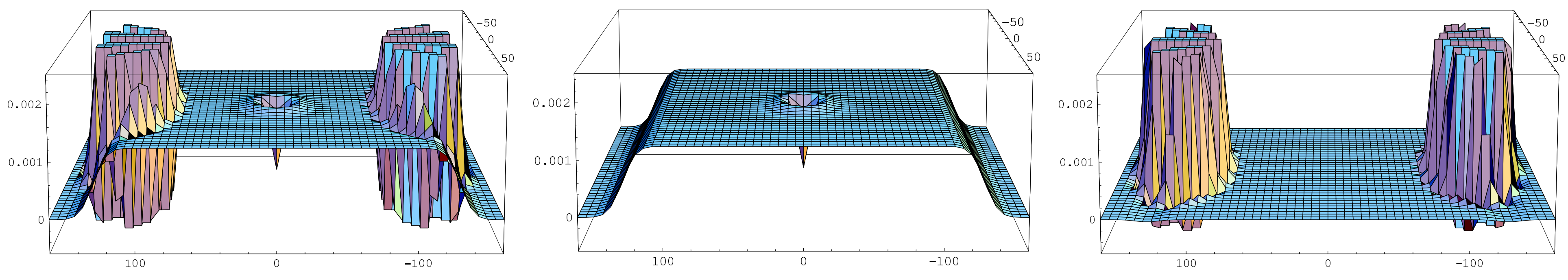
The chiral condensate from the eigenvalue density

$$\begin{aligned}\langle\bar{\psi}\psi\rangle(m) &= \frac{1}{V}\partial_m \log Z(m;\mu) \\ &= \frac{1}{V}\int dxdy\,\rho(x,y)\frac{1}{x+iy+m}\end{aligned}$$

The oscillations of the density are responsible for chiral symmetry breaking

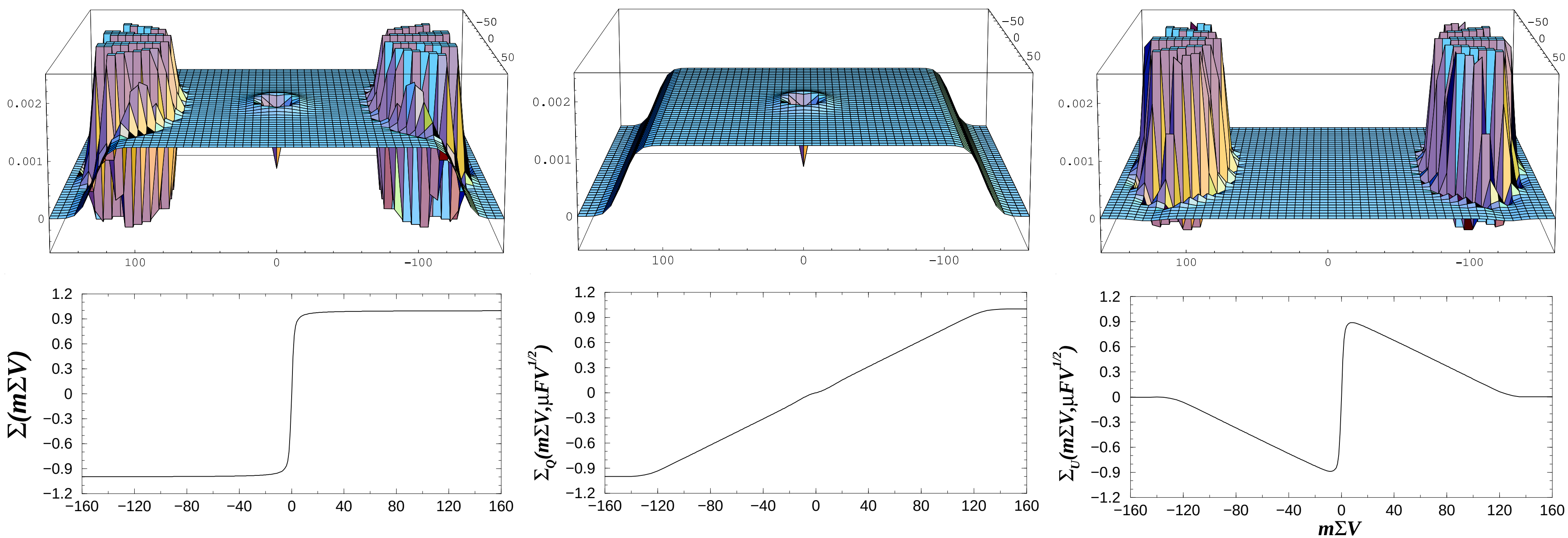
Osborn Splittorff Verbaarschot PRL 94 (2005) 202001

The unquenched eigenvalue density



Structure: $\rho_{N_f=1} = \rho_Q + \rho_U$

The unquenched chiral condensate



Structure: $\langle\bar{\psi}\psi\rangle_{N_f=1}(m) = \langle\bar{\psi}\psi\rangle_Q(m) + \langle\bar{\psi}\psi\rangle_U(m)$

Banks-Casher $\mu = 0$

$\boxed{\text{Accumulation of eigenvalues on the } y\text{-axis is responsible for chiral symmetry breaking}}$

OSV $\mu \neq 0$

$\boxed{\text{The oscillations of the eigenvalue density are responsible for chiral symmetry breaking}}$

Observation: The complex oscillations of the spectral correlation functions take part on the microscopic scale: period $\sim 1/V$ amplitude $\sim \exp(V)$

Exact microscopic result

Exact microscopic partition function and condensate

$$Z_{N_f=1}(m) = I_\nu(m) \qquad \langle\bar\psi\psi\rangle_{N_f=1}(m) = \frac{I'_\nu(m)}{I_\nu(m)}$$

$$\langle\bar\psi\psi\rangle_{N_f}(m) = \int dxdy\, \frac{\rho_{N_f}(x,y)}{x+iy-m}$$

Exact microscopic result

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Exact microscopic eigenvalue density

$$\begin{aligned}\rho^{(\nu)}_{N_f=1}(\hat z,\hat z^*) \quad &= \quad \frac{|\hat z|^2}{2\pi\hat\mu^2} K_\nu\left(\frac{|\hat z|^2}{4\hat\mu^2}\right) \mathrm{e}^{-\frac{\hat z^2+\hat z^{*\,2}}{8\hat\mu^2}} \\ &\times \left(\int_0^1 dt\; t\; \mathrm{e}^{-2\hat\mu^2 t^2} I_\nu(\hat z t) I_\nu(\hat z^* t) - \frac{I_\nu(\hat z)}{I_\nu(\hat m)} \int_0^1 dt\; t\; \mathrm{e}^{-2\hat\mu^2 t^2} I_\nu(\hat m t) I_\nu(\hat z^* t)\right)\end{aligned}$$

Osborn Splittorff Verbaarschot arXiv:0805.1303

Chiral Random Matrix Theory

The partition function - eigenvalue representation

$$\mathcal{Z}_N^{N_f}(m;\mu) = \int \prod_{k=1}^{N} d^2 z_k \left|\Delta_N(\{z_l^2\})\right|^2 |z_k|^{2\nu+2}$$
$$\times K_\nu\left(\frac{N(1+\mu^2)}{2\mu^2}|z_k|^2\right) e^{-\frac{N(1-\mu^2)}{4\mu^2}(z_k^2+{z_k^*}^2)}(m^2-z_k^2)^{N_f}$$

Osborn PRL 93 (2004) 222001

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The complex orthogonal polynomial method

$$\mathcal{Z}_N^{N_f=1}(m;\mu) = m^\nu p_N(m;\mu)$$

Osborn PRL 93 (2004) 222001

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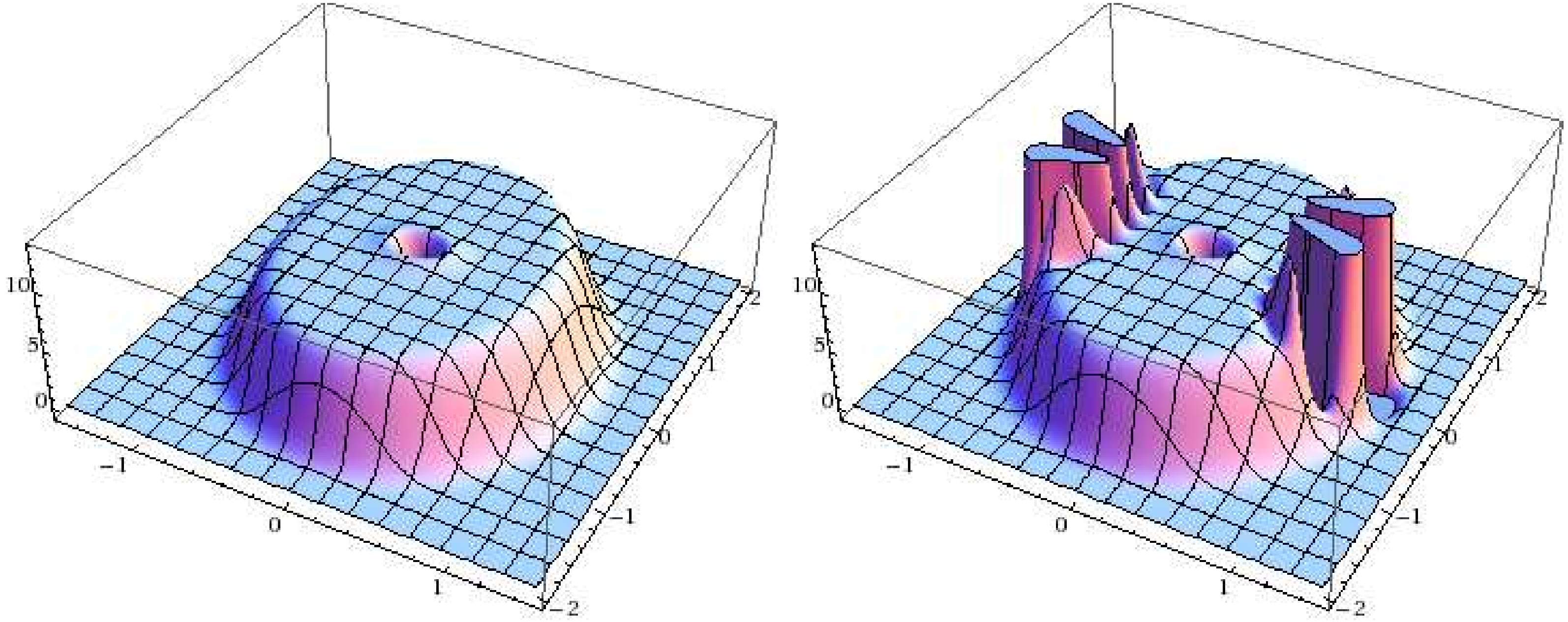
Osborn PRL 93 (2004) 222001

OSV relation at finite N

$$\mathcal{Z}_N^{N_f=1}(m) \quad = \quad m^\nu p_N(m) \qquad\qquad \langle\bar\psi\psi\rangle_N^{N_f=1}(m) = \frac{dp_N(m)/dm}{p_N(m)} + \frac{\nu}{m}$$

The eigenvalue density at $N = 20$



Osborn Splittorff Verbaarschot arXiv:0805.1303

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The eigenvalue density at finite N from the orthogonal polynomials

$$\rho_N^{N_f=1} = 2w(z, z^*; \mu) \sum_{k=0}^{N-1} \frac{p_k(z^*)(p_k(z) - p_N(z)p_k(m)/p_N(m))}{r_k}$$

Osborn Splittorff Verbaarschot arXiv:0805.1303, to appear in PRD

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The integral

$$\langle\bar{\psi}\psi\rangle_N^{N_f=1}(m) \quad = \quad \int dxdy \frac{\rho_N^{N_f=1}(x,y)}{x+iy+m}$$

Osborn Splittorff Verbaarschot arXiv:0805.1303, to appear in PRD

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Osborn Splittorff Verbaarschot arXiv:0805.1303, to appear in PRD

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can be done *using the orthogonality of the polynomials*

$$\int_{\mathbb{C}} d^2z\; w(z,z^*;\mu)\; p_k(z;\mu)\; p_l(z;\mu)^* \;\; = \;\; \delta_{kl}\; r_k^\nu$$

Osborn Splittorff Verbaarschot arXiv:0805.1303

Alternative way to compute the spectral density

The replica method

The replica way of writing the eigenvalue density

$$\rho^{N_f}(z, z^*, m; \mu) = \lim_{n \to 0} \frac{1}{\pi n} \partial_{z^*} \partial_z \log \mathcal{Z}_{N_f,n}(m, z, z^*; \mu)$$

generating functionals for the eigenvalue density

$$\mathcal{Z}_{N_f,n}(m, z, z^*; \mu) = \int \mathrm{d}A \det(D_\eta \gamma_\eta + \mu\gamma_0 + m)^{N_f} |\det(D_\eta \gamma_\eta + \mu\gamma_0 + z)|^{2n} \, \mathrm{e}^{-S_{\mathrm{YM}}(A)}$$

Girko Theory of Random Determinants

The Toda Lattice Equation

$$\partial_z \partial_{z^*} \log Z_{N_f,n} = 4zz^* n \frac{Z_{N_f,n+1} Z_{N_f,n-1}}{[Z_{N_f,n}]^2}$$

Verbaarschot, Zirnbauer, J. Phys. A **18**, 1093 (1985)
Kamenev Mézard J.Phys.A **32** 4373 (1999); PRB **60** 3944 (1999)
Yurkevich, Lerner, PRB **60**, 3955 (1999)
M.R. Zirnbauer, cond-mat/9903338
Kanzieper, PRL **89**, 250201 (2002)
Splittorff, Verbaarschot, PRL **90**, 041601 (2003)
Splittorff, Verbaarschot, Nucl.Phys. B683 (2004) 467

Akemann Osborn Splittorff Verbaarschot NPB 712 (2005) 287

The Toda Lattice Equation

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The Replica Limit ($n \to 0$) of the Toda lattice equation $n \to 0$

$$\rho_{N_f}(z, z^*, m; \mu) = 4zz^* \frac{Z_{N_f,n=1}(m, z, z^*; \mu) Z_{N_f,n=-1}(m|z, z^*; \mu)}{[Z_{N_f}(m; \mu)]^2}$$

Verbaarschot, Zirnbauer, J. Phys. A **18**, 1093 (1985)
Kamenev Mézard J.Phys.A **32** 4373 (1999); PRB **60** 3944 (1999)
Yurkevich, Lerner, PRB **60**, 3955 (1999)
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Bosonic quarks = average inverse determinants

$$Z_{N_f=-1} = \left\langle \frac{1}{\det(D + \mu\gamma_0 + m)} \right\rangle$$

From Cauchy transform of orthogonal polynomials

$$Z_{N_f=-1} = -\frac{1}{r_{N-1}} m^{-\nu} \int d^2 z w(z, z^*; \mu) p_{N-1}(z)^* \frac{1}{z^2 - m^2}$$

Akemann, Pottier, J.Phys. A37 (2004) 453

Bergère arXiv:hep-th/0404126

Feinberg Zee NPB 504 (1997) 578

Splittorff, Verbaarschot Nucl.Phys. B757 (2006) 259

Splittorff, Verbaarschot, Zirnbauer arXiv 0802.2660

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$$Z_{N_f=-1} = \left\langle \frac{1}{\det(D + \mu\gamma_0 + m)} \right\rangle$$

From Cauchy transform of orthogonal polynomials

$$Z_{N_f=-1} = -\frac{1}{r_{N-1}} m^{-\nu} \int d^2 z w(z, z^*; \mu) p_{N-1}(z)^* \frac{1}{z^2 - m^2}$$

From σ-model: Because of convergence requirements

$$Z_{N_f=-1} = \left\langle \frac{\det(D + \mu\gamma_0 + m)^*}{\det \begin{pmatrix} \epsilon & D + \mu\gamma_0 + m \\ (D + \mu\gamma_0 + m)^* & \epsilon \end{pmatrix}} \right\rangle$$

Akemann, Pottier, J.Phys. A37 (2004) 453
Bergère arXiv:hep-th/0404126
Feinberg Zee NPB 504 (1997) 578
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Conclusions

Eigenvalue density of Non Hermitian chRMT is complex valued

Chiral symmetry breaking linked to oscillations at the microscopic scale

Shows the numerical difficulties in dealing with the sign problem

At finite N cancellations are due to orthogonality of the polynomials

Replica limit of the Toda Lattice equation as alternative to OP

Averages of inverse determinants of non hermitian operators

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Additional slides

Central observation

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$$Z_{N_f,n} = \int_{U(N_f+2n)} dU e^{-\frac{V}{4}F_\pi^2\mu^2 \mathrm{Tr}[U,B][U^{-1},B] \; + \frac{1}{2}m\langle\bar{\psi}\psi\rangle V \mathrm{Tr}(U+U^{-1})}$$